MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

EME4076 – MECHANICAL VIBRATIONS

(All sections)

09 MARCH 2016
02:30 p.m. – 04:30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 9 pages including cover page with 5 Questions and 3 appendices.

2. Attempt **FOUR** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

As shown in Figure Q1, a small 8-kg sphere B is attached to the rigid bar AB of negligible mass that is hinged at A by a pin. A moving support D is connected to the bar at C by means of a spring of stiffness $k = 20$ kN/m. However, a dash-pot with a viscous resistance of c is used to connect to the bar at B. The support D is subjected to a vertical sinusoidal displacement motion of $\delta = \delta_m \sin \omega t$ with amplitude $\delta_m = 1.25$ mm and frequency ω.

(a) If $\omega = 3$ Hz and $c = 35$ N-s/m, determine the amplitude of the resulting steady motion of the sphere B and the maximum dynamic force exerted by the spring on the support D.

[15 marks]

(b) If $c = 0$ N-s/m, determine the range of ω for which the amplitude of fluctuating force exerted by the spring on the bar is less than 25 N for the cases that the motions of the bar AB and the support D are (i) in phase, and (ii) out of phase.

[10 marks]

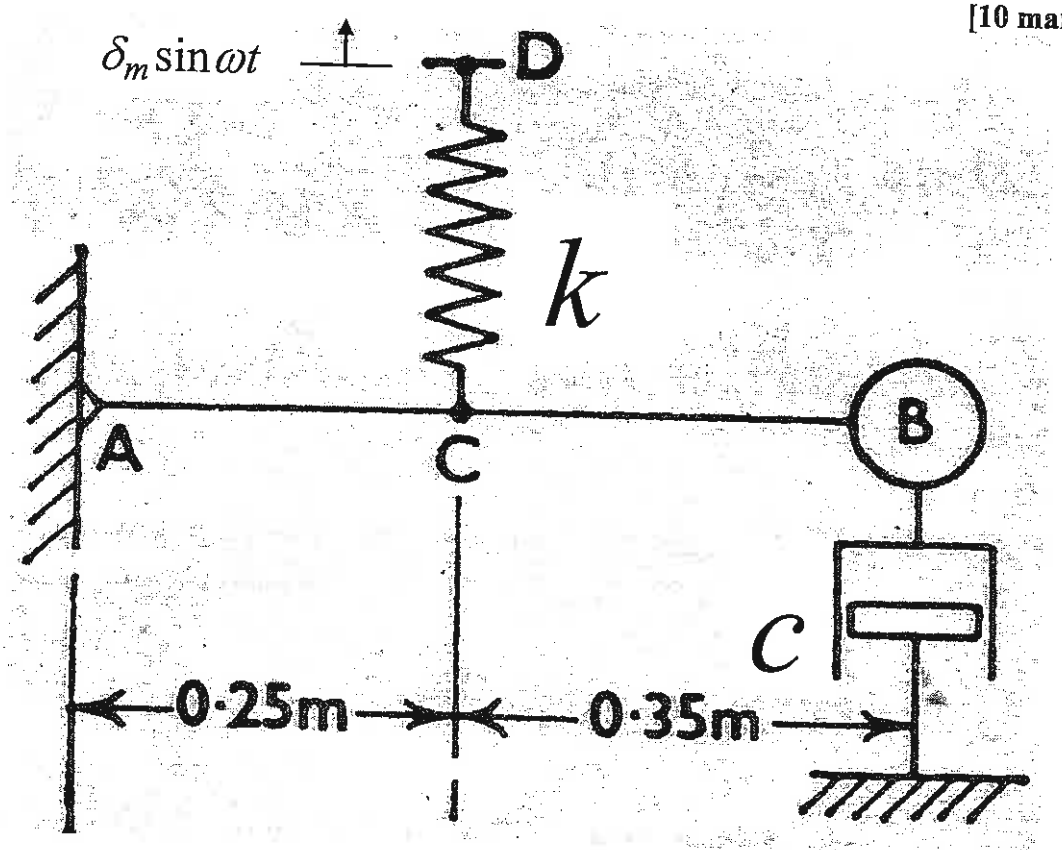


Figure Q1

Continued .....

Question 2

Figure Q2 shows the top view of a simplified washing machine model. A bundle of wet clothes forms a rotating unbalance mass of m_b in the machine.

(a) As the rotating speed of the machine increases to full speed, measurements of the displacement are made. It is noted that at resonance the amplitude of the displacement of the machine is 10 mm. As the spin speed increases significantly above resonance, the displacement is observed to be 1 mm. Determine the damping ratio of the system.

[10 marks]

(b) The total mass of the machine including the wet clothes is 20 kg. The diameter of the washer basket ($2e$) is 50 cm. The mass of the wet clothes is 10 kg. The full speed of the machine is 300 rpm. At full speed the maximum force allowed to transmit to the side of the washing machine (considered as foundation) is 100 N. Design the isolation system (i.e. determine the values of k and c to use) at full speed.

[15 marks]

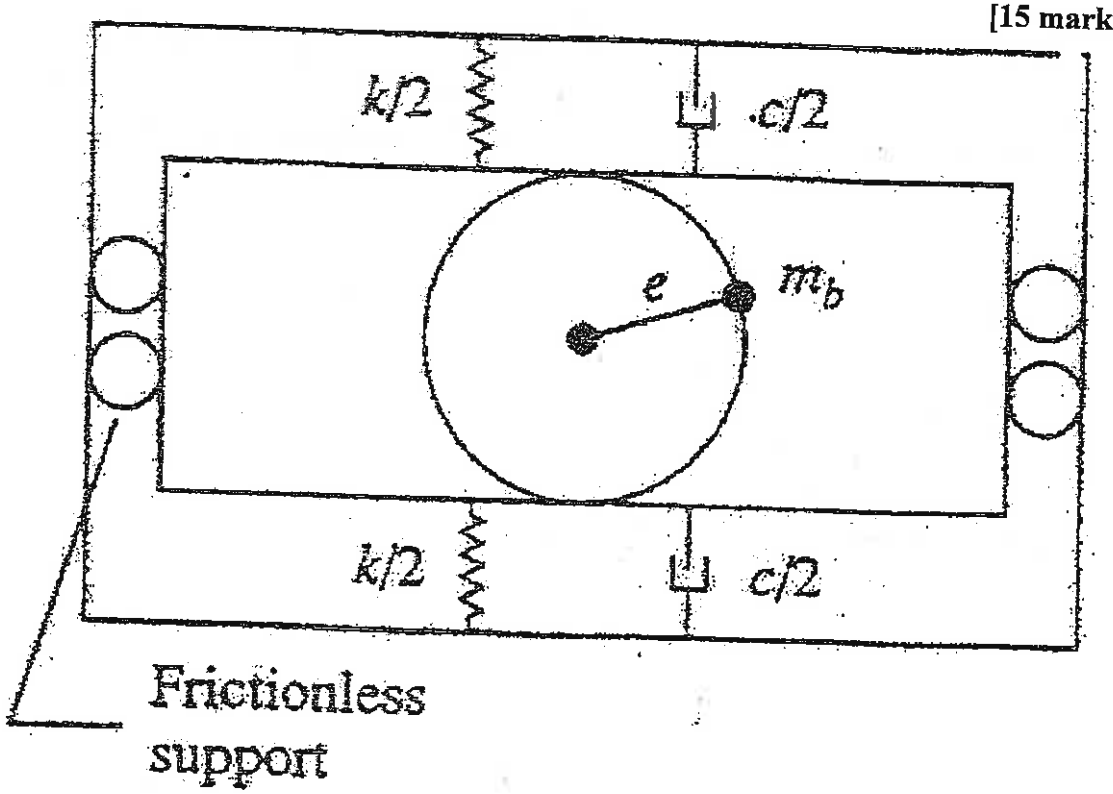


Figure Q2

Continued .....

Question 3

Figure Q3 shows a tractor and trailer system which is used to haul large cylindrical drum (paper, pipe, etc.) around an industrial plant. As shown in the figure, the trailer carries a heavy cylindrical drum of mass m and radius r. The drum can roll without slipping on the trailer. Both the tractor/trailer coupling and trailer/drum attachment have a stiffness k. Given that the masses of the tractor and trailer are $10m$ and $2m$, respective. At the instance shown in the figure, the system is at its static equilibrium. If the trailer is subjected to the sinusoidal force $F = F_0 \sin \omega t$, determine

(a) the differential equations of the system in terms of x_1 and x_2 where x_1 and x_2 are the displacements of the trailer and center of the drum, respectively. Since the mass of the tractor is much larger than the trailer, the tractor can be treated as a fixed support,

[12 marks]

(b) the steady-state response amplitudes X_1 and X_2 of the trailer and drum, respectively, and

[7 marks]

(c) the conditions under which (i) the drum will act as a vibration absorber for the trailer such that $X_1 = 0$ and (ii) the drum will not oscillate.

[6 marks]

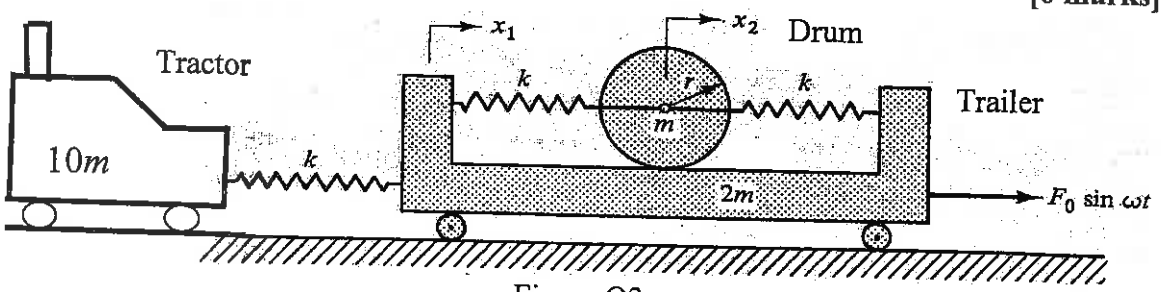


Figure Q3

Continued .....

Question 4

Figure Q4 shows a bullet of mass $m = 6\,\text{g}$ is fired into a wood block of mass $M = 1\,\text{kg}$ which is at equilibrium at the instance shown. The block is suspended by an inextensible cord with a length $l = 1.2\,\text{m}$. After firing, the bullet embedded block swings to position A which is making an angle $\theta = 5^0$ with vertical. Due to gravitational force, the bullet embedded block then swings backwards from position A first about O and then about the peg P after the cord comes in contact with the peg. Neglecting the energy lost during the impact and aerodynamic drag, determine

(a) the time taken for the bullet embedded block measured from position A to swing one cycle,

[10 marks]

(b) the maximum angle that the bullet embedded block will swing to the other side after the cord hits the peg P, and

[10 marks]

(c) the tension in the cord 0.25 second after the bullet embedded block swings backwards from its position A.

[5 marks]

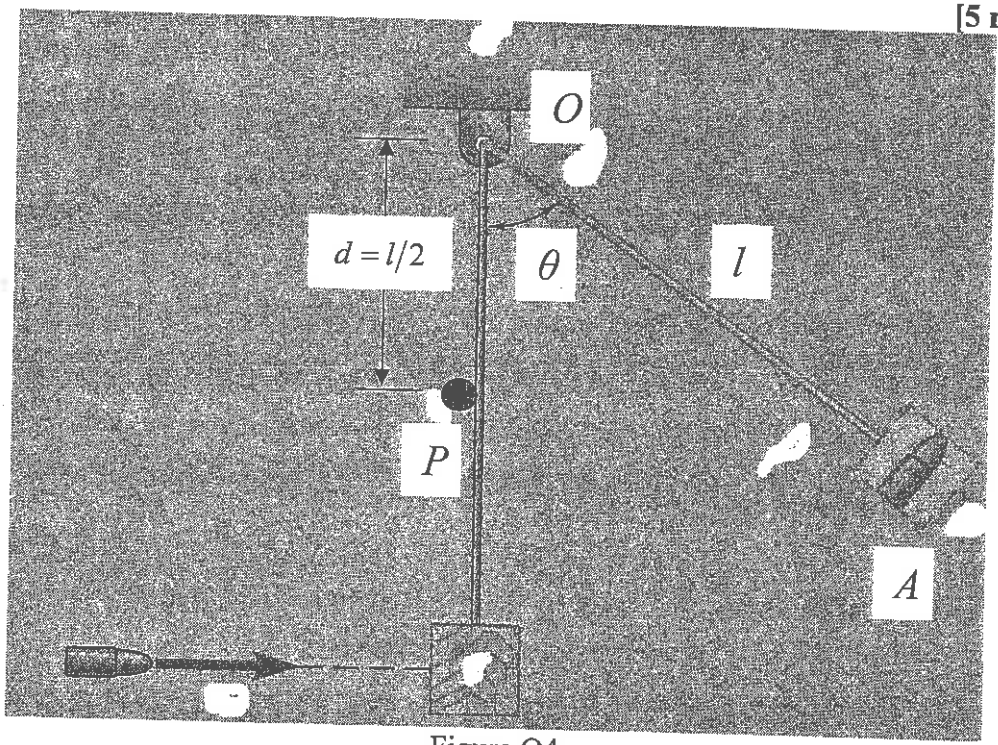


Figure Q4

Continued .....

Question 5

Figure Q5(i) shows a quarter car model of an automobile which is moving at a constant speed of v_0 towards a guardhouse of a guarded residential area. Figure Q5(ii) shows the speed bumps before the guardhouse which are used to force the driver of the car to slow down as he/she approaches the guardhouse. The bumps can be described as half sine curve $Y_0 \sin(\pi t/t_0)$.

(a) Determine the width of the first strip such that the response of the car $x(t)$ in between the first and second half sine curve is maximized.
Hint: use the trigonometric identity $\sin(a)\sin(b) = 1/2(\cos(a-b) - \cos(a+b))$.
[20 marks]

(b) Locate the second half sine curve such that the response of the car after the time $t_1 + t_0$ is minimized that is $x(t) = 0$.
[5 marks]

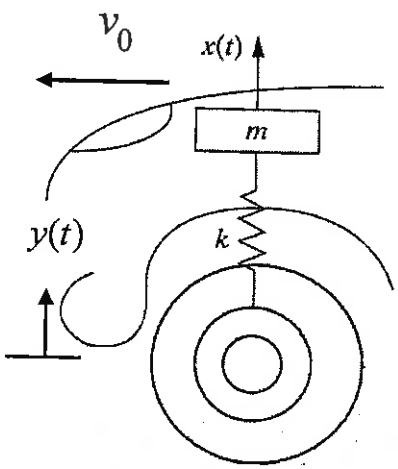


Figure Q5(i)

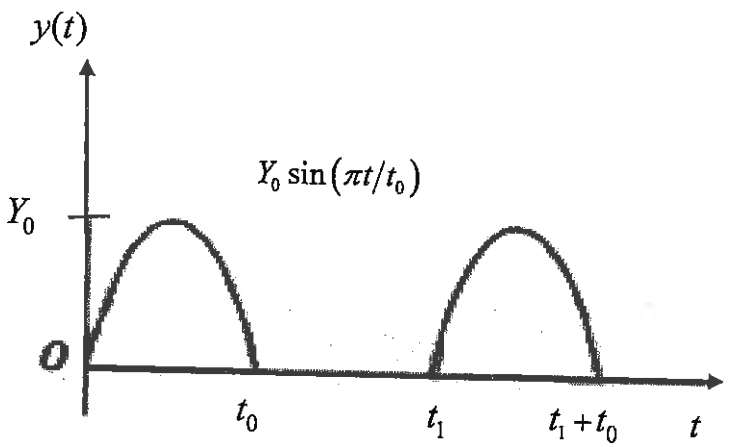


Figure Q5(ii)

Continued .....

Appendix 1

Vibrations

Equations of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D \sin(\omega t - \varphi)$$

$$D = \frac{F_0/k}{\sqrt{[1-r^2]^2 + (2\zeta r)^2}} \qquad where \quad r = \frac{\omega}{\omega_n}$$

$$\varphi = \tan^{-1} \frac{2\zeta r}{1-r^2}$$

$$x_c(t) = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} \qquad \zeta > 1$$

$$x_c(t) = (B + Ct) e^{-\omega_n t} \qquad \zeta = 1$$

$$x_c(t) = e^{-\zeta \omega_n t} (B \cos \omega_d t + C \sin \omega_d t) \qquad \zeta < 1$$

$$\omega_n = \sqrt{k/m} = 2\pi f_n, \qquad f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\lambda_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \qquad \zeta \geq 1$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \qquad \zeta < 1$$

Force and Displacement Transmissibility

$$\frac{F_T}{F_0} = \frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}}$$

Rotating Unbalance: Amplitude of Response

$$X = \frac{m_0 e}{m + m_0} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Fourier's series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

$$a_n = \frac{2}{\tau}\int_0^{\tau} x(t) \cos n\omega t \, dt \qquad b_n = \frac{2}{\tau}\int_0^{\tau} x(t) \sin n\omega t \, dt$$

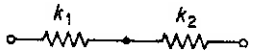
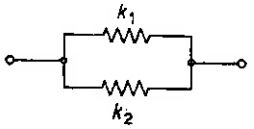
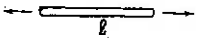
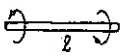
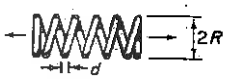
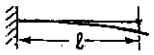
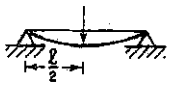
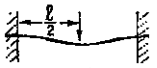
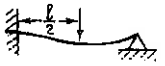
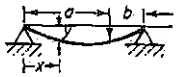
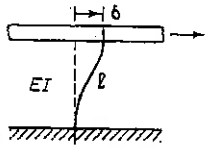
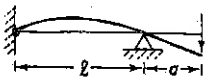
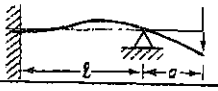
Unit response function (underdamped)

$$h(t-\tau) = \frac{e^{-\xi\omega_n(t-\tau)}}{m\omega_d} \sin \omega_d (t-\tau)$$

Continued .....

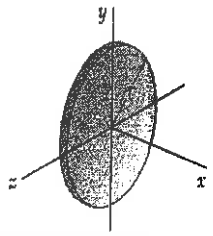
Appendix 2

Table of Spring Stiffness

	$k = \frac{1}{1/k_1 + 1/k_2}$	
	$k = k_1 + k_2$	
	$k = \frac{EI}{l}$	I = moment of inertia of cross-sectional area l = total length
	$k = \frac{EA}{l}$	A = cross-sectional area
	$k = \frac{GJ}{l}$	J = torsion constant of cross section
	$k = \frac{Gd^4}{64nR^3}$	n = number of turns
	$k = \frac{3EI}{l^3}$	k at position of load
	$k = \frac{48EI}{l^3}$	
	$k = \frac{192EI}{l^3}$	
	$k = \frac{768EI}{7l^3}$	
	$k = \frac{3EIl}{a^2b^2}$	$y_x = \frac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \frac{12EI}{l^3}$	
	$k = \frac{3EI}{(l + a)a^2}$	
	$k = \frac{24EI}{a^2(3l + 8a)}$	

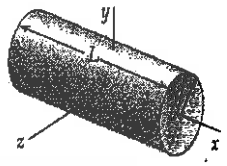
Continued .....

Appendix 3: Mass Moment of Inertia of Homogenous Solids



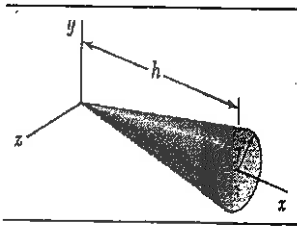
$I_x = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$



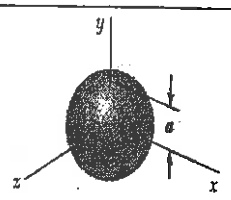
$I_x = \frac{1}{2}ma^2$

$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$

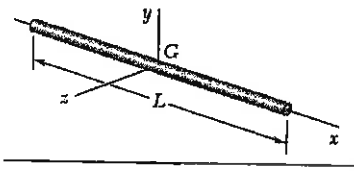


$I_x = \frac{3}{10}ma^2$

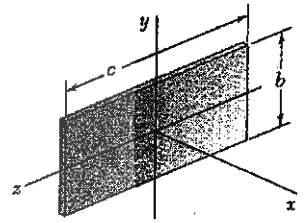
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



$I_x = I_y = I_z = \frac{2}{5}ma^2$



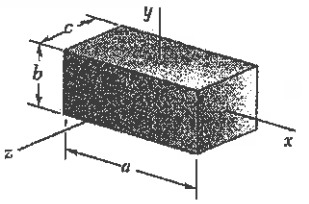
$I_y = I_z = \frac{1}{12}mL^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}mc^2$

$I_z = \frac{1}{12}mb^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}m(c^2 + a^2)$

$I_z = \frac{1}{12}m(a^2 + b^2)$

End of Paper